Improvements on computation of phase velocities of Rayleigh waves based on the generalized R/T coefficient method

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ABSTRACT

Among many improvements on phase velocity computation of Rayleigh waves in a layered earth model, the methods based on R/T (reflection and transmission) coefficients produce stable and accurate phase velocities for both low and high frequencies and are appropriate for viscoelastic and anisotropic media. The generalized R/T coefficient methods are not the most efficient algorithms. This paper presents a new, more efficient algorithm, called the fast generalized R/T coefficient method, to calculate the phase velocity of surface waves for a layered earth model. The improvements include 1) computation of the generalized R/T coefficients without calculation of the modified R/T coefficients; 2) presenting an analytic solution for the inverse of the 4X4 layer matrix **E**. Compared with the traditional generalized R/T coefficient method, the fast generalized R/T coefficient method, when applied on Rayleigh waves, significantly improves the speed of computation, cutting the computational time at least by half while keeping the stability and accuracy of the traditional method.

INTRODUCTION

Rayleigh waves are dispersive over layered geologies, that is, the phase velocity of a Rayleigh wave depends on its frequency (the relation is called a dispersion curve). Longer wavelength Rayleigh waves penetrate deeper than shorter wavelengths for a given mode and generally have greater phase velocities. The phase velocities of surface waves have to be calculated for many applications, for example in modeling regional L_g and R_g waves (Oliver and Ewing, 1957), in inferring earth structures (Brune and

Dorman, 1963), as well as in synthesizing the complete seismogram (Zeng and Anderson, 1995).

Methods used to calculate surface wave dispersion curves for a flat-layered earth model begin with Thomson and Haskell (Thomson, 1950; Haskell, 1953) who used a matrix to solve the eigenvalue problem of the system of differential equations. The original Thomson-Haskell formalism was unstable and had many numerical difficulties that are associated with numerical overflow and loss of precision at high frequencies (Kennett, 1983; Buchen and Ben-Hador, 1996). Later improvements on the formalism are to overcome these problems, including the delta matrix (Pestel and Leckie, 1963), reduced delta matrix (Watson, 1970), fast delta matrix (Buchen and Ben-Hador, 1996), Schwab-Knopoff method (Schwab and Knopoff, 1970, 1972), fast Schwab-Knopoff (Schwab, 1970), Abo-Zena method (Abo-Zena, 1979), and Kennett R/T (reflection and transmission) matrix (Kennett, 1974; Kennett and Kerry, 1979), and the generalized R/T coefficient method (Kennett, 1983; Luco and Apsel, 1983),

The R/T methods are the least efficient of these improved methods mentioned above (Buchen and Ben-Hador, 1996). However, they are numerically more stable for high frequency cases (Chen, 1993). Phase velocities over 100 Hz for a layered crustal model are calculated (Chen, 1993). Plus for a lateral heterogeneous and viscoelastic media, the R/T method is the most stable algorithm for computing the phase velocity dispersion curves (Buchen and Ben-Hador, 1996).

The generalized R/T coefficient method was introduced by Kennett (1983) and Luco and Apsel (1983), and later improved by Chen (1993) and Hisada (1994, 1995). Both Chen's and Hisada's versions produce stable and accurate phase velocities for both low and high frequencies. However, both versions are not the most efficient algorithm (Buchen and Ben-Hador, 1996). This paper presents a new more efficient algorithm, called the fast generalized R/T coefficient method, to calculate the phase velocity of surface waves for a layered earth model. The fast method is based on but is more efficient than the method of Chen (1993) and Hisada (1994, 1995).

In this paper, we first briefly summarize basic algorithm of the generalized R/T coefficient method of Chen (1993) for Rayleigh waves, followed by our improvements. Then we test our version at both crustal and local site scales.

METHOD

Basic Theory

Consider a plane surface wave in a horizontally layered, vertically heterogeneous, isotropic, elastic medium over a half-space (Figure 1). Elastic moduli μ_j λ_j ρ_j (rigidity, Lame's parameter, and density of the jth layer, respectively) are dependent on depth and are constant within layers. The differential equations for motion-stress vectors of Rayleigh waves (Aki and Richards, 2002, equation (7.28)) can be obtained by solving the elastodynamic equations (without the source term) with the free surface boundary conditions, continuity of the wave field across each interface, and radiation condition at infinity. Among many other methods to solve the differential equations, we favor the generalized R/T coefficient method (Kennett, 1983; Luco and Apsel, 1983; Chen, 1993; Hisada, 1994, 1995). This method naturally excludes the growth exponential terms during matrix multiplication and thus yields a more stable and comprehensive numerical procedure for computation of surface-wave dispersion curves.

Rayleigh waves consist of P and SV-waves. According to Chen (1993), the general solution of the differential equations for motion-stress vectors of Rayleigh waves within a homogeneous layer can be expressed as product of layer matrix **E**, phase delay matrix **Λ**, and amplitude vector matrix **C**. It is given as (see Appendix A for each term)

$$\begin{bmatrix} \mathbf{D}^{j}(z) \\ \mathbf{S}^{j}(z) \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^{j} & \mathbf{E}_{12}^{j} \\ \mathbf{E}_{21}^{j} & \mathbf{E}_{22}^{j} \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_{d}^{j}(z) & 0 \\ 0 & \mathbf{\Lambda}_{u}^{j}(z) \end{bmatrix} \begin{bmatrix} \mathbf{C}_{d}^{j} \\ \mathbf{C}_{u}^{j} \end{bmatrix} = \mathbf{E}^{j}\mathbf{\Lambda}^{j}\mathbf{C}^{j} \qquad (1)$$

For an arbitrary jth interface, the modified reflection and transmission coefficients for Rayleigh waves are denoted as $(\mathbf{R}_{du}^{j}, \mathbf{R}_{ud}^{j}, \mathbf{T}_{d}^{j}, \mathbf{T}_{u}^{j})$ and defined by the following equations:

$$\begin{cases} \mathbf{C}_{d}^{j+1} = \mathbf{T}_{d}^{j}\mathbf{C}_{d}^{j} + \mathbf{R}_{ud}^{j}\mathbf{C}_{u}^{j+1} \\ \mathbf{C}_{u}^{j} = \mathbf{R}_{du}^{j}\mathbf{C}_{d}^{j} + \mathbf{T}_{u}^{j}\mathbf{C}_{u}^{j+1} \end{cases} \qquad (2a)$$

for $j = 1, 2, 3, \cdots, N-1$ and

$$\begin{cases} \mathbf{C}_{d}^{N+1} = \mathbf{T}_{d}^{N}\mathbf{C}_{d}^{N} \\ \mathbf{C}_{u}^{N} = \mathbf{R}_{du}^{N}\mathbf{C}_{d}^{N} \end{cases} \qquad (2b)$$

for $j = N$

where $\mathbf{R}_{du}^{j} = \begin{bmatrix} R_{dpp}^{j} & R_{dsp}^{j} \\ R_{dps}^{j} & R_{dss}^{j} \end{bmatrix}$, $\mathbf{R}_{ud}^{j} = \begin{bmatrix} R_{upp}^{j} & R_{usp}^{j} \\ R_{ups}^{j} & R_{uss}^{j} \end{bmatrix}$, $\mathbf{T}_{d}^{j} = \begin{bmatrix} T_{dpp}^{j} & T_{dsp}^{j} \\ T_{dps}^{j} & T_{dss}^{j} \end{bmatrix}$, and $\mathbf{T}_{u}^{j} = \begin{bmatrix} T_{upp}^{j} & T_{usp}^{j} \\ T_{ups}^{j} & T_{uss}^{j} \end{bmatrix}$. Sub-index 'd' means down-going waves; 'u' up-going waves; 'p' P-waves; and 's' SV-waves. R_{dps}^{j} is the reflection coefficient of incident down-going P-wave to reflected SV-wave at interface j. T_{dps}^{j} is the transmission coefficient of incident down-going P-wave to transmitted down-going SV-wave at interface j. Other terms have the similar physical meaning. After applying the continuity condition at each interface, we obtain the explicit expressions of the modified R/T coefficient matrices as follows:

$$\begin{bmatrix} \mathbf{T}_d^j & \mathbf{R}_{ud}^j \\ \mathbf{R}_{du}^j & \mathbf{T}_u^j \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^{j+1} & -\mathbf{E}_{12}^j \\ \mathbf{E}_{21}^{j+1} & -\mathbf{E}_{22}^j \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{E}_{11}^j & -\mathbf{E}_{12}^{j+1} \\ \mathbf{E}_{21}^j & -\mathbf{E}_{22}^{j+1} \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_d^j(z^j) & 0 \\ 0 & \mathbf{\Lambda}_u^{j+1}(z^j) \end{bmatrix} \qquad (3a)$$

for $j = 1, 2, 3, \cdots, N-1$

$$\begin{bmatrix} \mathbf{T}_d^N & \mathbf{R}_{ud}^N \\ \mathbf{R}_{du}^N & \mathbf{T}_u^N \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^{N+1} & -\mathbf{E}_{12}^N \\ \mathbf{E}_{21}^{N+1} & -\mathbf{E}_{22}^N \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{E}_{11}^j \mathbf{\Lambda}_d^j(z^j) \\ \mathbf{E}_{21}^j \mathbf{\Lambda}_d^j(z^j) \end{bmatrix} \qquad (3b)$$

for $j = N$

Note that the layer matrix $\mathbf{E}$ is composed of elements that is determined by the elastic parameters of both jth and (j+1)th layers.

For an arbitrary jth interface, the generalized R/T coefficients for Rayleigh waves are denoted as $(\hat{\mathbf{R}}_{du}^j, \hat{\mathbf{T}}_d^j)$ and defined by the following equations:

$$\begin{cases} \mathbf{C}_d^{j+1} = \hat{\mathbf{T}}_d^j \mathbf{C}_d^j \\ \mathbf{C}_u^j = \hat{\mathbf{R}}_{du}^j \mathbf{C}_d^j \end{cases} \qquad (4a)$$

for $j = 1, 2, 3, \cdots, N-1$

$$\begin{cases} \mathbf{C}_d^{N+1} = \hat{\mathbf{T}}_d^N \mathbf{C}_d^N \\ \hat{\mathbf{R}}_{du}^N = \mathbf{0} \end{cases} \qquad (4b)$$

for $j = N$

Comparing equation (2) and (4) we find

$$\hat{\mathbf{T}}_d^N = \mathbf{T}_d^N \quad \textit{and} \quad \hat{\mathbf{R}}_{du}^N = \mathbf{R}_{du}^N \qquad (5).$$

Substituting equation (4) in equation (2), we obtain the recursive formula for computing other generalized R/T coefficients as

$$\begin{cases} \hat{\mathbf{T}}_d^j = (\mathbf{I} - \mathbf{R}_{ud}^j \hat{\mathbf{R}}_{du}^{j+1})^{-1} \mathbf{T}_d^j \\ \hat{\mathbf{R}}_{du}^j = \mathbf{R}_{ud}^j + \mathbf{T}_u^j \hat{\mathbf{R}}_{du}^{j+1} \hat{\mathbf{T}}_d^j \end{cases} \qquad (6)$$

for $j = 1, 2, 3, \cdots, N-1$

Starting from the last interface where $\hat{\mathbf{R}}_{du}^N = \mathbf{0}$ we can use equation (6) to find the generalized R/T coefficients ($\hat{\mathbf{R}}_{du}^j, \hat{\mathbf{T}}_d^j$) for Rayleigh waves for all interfaces above.

The Rayleigh modes can be determined by imposing the traction-free condition at the free surface (z=0). From equation (1) we calculate the traction at the free surface as

$$\mathbf{S}^1(0) = (\mathbf{E}_{21}^1 + \mathbf{E}_{22}^1 \mathbf{\Lambda}_u^0(0) \hat{\mathbf{R}}_{du}^1) \mathbf{C}_d^1 \qquad (7)$$

Equation (7) has non-trivial solutions only for some particular phase velocities that satisfy the following secular equation:

$$\det(\mathbf{E}_{21}^1 + \mathbf{E}_{22}^1 \mathbf{\Lambda}_u^0(0) \hat{\mathbf{R}}_{du}^1) = 0 \qquad (8)$$

Equation (8) is called the secular function for Rayleigh waves. Therefore, the roots of this equation are the phase velocities for modes that potentially exist.

Improvements

Careful observation from equation (8) reveals that only the generalized R/T coefficients are needed to calculate the secular function of Rayleigh waves. In fact, the generalized R/T coefficients could be directly calculated without knowing the modified R/T coefficients. Therefore, the modified R/T coefficients are not necessary for the calculation of the secular function of Rayleigh waves (equation (8)).

The continuity condition at any arbitrary interface j states that

$$\begin{bmatrix} \mathbf{E}_{11}^j & \mathbf{E}_{12}^j \\ \mathbf{E}_{21}^j & \mathbf{E}_{22}^j \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_d^j(z^j) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{C}_d^j \\ \mathbf{C}_u^j \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^{j+1} & \mathbf{E}_{12}^{j+1} \\ \mathbf{E}_{21}^{j+1} & \mathbf{E}_{22}^{j+1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \mathbf{\Lambda}_u^{j+1}(z^j) \end{bmatrix} \begin{bmatrix} \mathbf{C}_d^{j+1} \\ \mathbf{C}_u^{j+1} \end{bmatrix} \qquad (9)$$

The definition of the generalized R/T coefficients implies that

$$\begin{cases} \mathbf{C}_u^j = \hat{\mathbf{R}}_{du}^j \mathbf{C}_d^j \\ \mathbf{C}_d^{j+1} = \hat{\mathbf{T}}_d^j \mathbf{C}_d^j \\ \mathbf{C}_u^{j+1} = \hat{\mathbf{R}}_{du}^{j+1} \hat{\mathbf{T}}_d^j \mathbf{C}_d^j \end{cases} \qquad (10)$$

Directly substituting equation (10) into equation (9) yields

$$\begin{bmatrix} \mathbf{I} \\ \hat{\mathbf{R}}_{du}^j \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{11}^j & \mathbf{E}_{12}^j \\ \mathbf{E}_{21}^j & \mathbf{E}_{22}^j \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{E}_{11}^{j+1} & \mathbf{E}_{12}^{j+1} \\ \mathbf{E}_{21}^{j+1} & \mathbf{E}_{22}^{j+1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{T}}_d^j \\ \mathbf{\Lambda}_u^{j+1}(z^j) \hat{\mathbf{R}}_{du}^{j+1} \hat{\mathbf{T}}_d^j \end{bmatrix} \qquad (11)$$

Starting from the last interface where $\hat{\mathbf{R}}_{du}^N = \mathbf{0}$, equation (11) yields the generalized R/T coefficients ($\hat{\mathbf{R}}_{du}^j, \hat{\mathbf{T}}_d^j$) of Rayleigh waves for all interfaces above.

Thus we can directly calculate the generalized R/T coefficients without knowing the modified R/T coefficients. The next step is to derive the inverse matrix of 4X4 **E** matrix in equation (11). The inverse of **E** in equation (3) is calculated by the elimination method given by Hisada (1995, equation (A1)). It is one of the most CPU-time consuming parts in the R/T method (Hisada, 1995). Unlike these of the layer matrix **E** in equation (3), the elements of the layer matrix **E** in equation (11) are determined by the elastic parameters of the jth layers only. This characteristic allows many terms to be crossed out during the derivation of the inverse matrix $\mathbf{E}^{-1}$ and results in a simple analytic solution for the inverse matrix $\mathbf{E}^{-1}$ (equation (A8) in Appendix A). For equation (3) it is impossible to derive the similar analytic form of $\mathbf{E}^{-1}$ as the elements of matrix **E** are related to the elastic parameters of both jth and (j+1)th layers. The simplicity of our solution significantly reduces the computational time. The following test section shows that these improvements cut the computational time for dispersion curves of Rayleigh waves at least by half.

NUMERICAL EXAMPLES

Three cases are designed for the large scale. Model 1 is Gutenberg's classic Earth model for a continent (Aki and Richards, 2002, p. #279), which resembles the velocity structure of the Earth to the depth of 1000 km. The model consists of a stack of 24 homogeneous and isotropic layers and has been used for many geophysical studies and provides an excellent reference. Model 2 is an artificial, inverted profile, which has a low velocity zone for the fourth layer (Table 1). Model 3 is another artificial four-layer profile, which has a high velocity zone for the second layer (Table 1). Three other cases are designed for the local scale. Model 4 is a regular stack of 4 homogeneous and isotropic layers (Table 2). Model 5 has a low velocity zone for the second layer and model 6 has a high velocity layer for the second layer (Table 2).

In the traditional version (Chen, 1993; Hisada, 1994, 1995), we first calculate the modified R/T coefficients (equation (3)) then the generalized R/T coefficients (equation (6)). The inverse matrix $\mathbf{E}^{-1}$ is computed following Hisada's procedure (1994). Using bisection root-searching method, the phase velocities could be found from the secular equation (8). We coded the above calculation in a program called RTmod, emphasizing the fact that the generalized R/T coefficients are based on the modified R/T coefficients. In the fast generalized R/T coefficients method, the generalized R/T coefficients are directly computed from equation (11) without calculations of the modified R/T coefficients. Keeping other parts identical, we code the fast generalized R/T coefficients method in a program called RTgen. We perform stability and efficiency tests for RTgen at both crustal and local site scales.

The stability tests of the fast generalized R/T coefficients method are done by comparing the calculated phase velocities for a given model, by RTgen and by CPS. CPS (Computer Programs in Seismology) is a software package developed by Herrmann and Ammon (2002), in which there are functions to calculate the phase velocity dispersion curves of surface waves. These functions of CPS have been widely used to compute the dispersion curves of Rayleigh waves (e.g., Stephenson et al., 2005). Figure 2 and 3 show the fundamental-mode dispersion curves of Rayleigh waves calculated by RTgen plotted atop those by CPS. The dispersion curves cover a broad frequency range from 0.01 Hz to 100 Hz. Both calculations yield identical results to 1%, indicating the fast generalized R/T coefficients method is stable and accurate. Higher-mode (up to mode 17) dispersion curves of Rayleigh waves for the six test models by both CPS and RTgen are almost the same, within 5% accuracy.

The efficiency tests of the fast generalized R/T coefficients method are done by comparing computational time taken by RTgen and RTmod for models with differing numbers of layers. We coded both RTmod and RTgen in an identical way except in how to calculate the generalized R/T coefficients. The coefficients are calculated from the modified R/T coefficients (equation (6)) in RTmod, and directly computed from equation (11) in RTgen.

Both codes ran on Linux Pentium machines, Sun workstations, personal Windows PCs, and OS X Macintosh machines. Figure 4 shows the computational times in seconds for 20 runs on a PowerPC G4 Mac notebook that has a 1.33 GHz processor.

The 24-layer-model used for efficiency test is Gutenberg's crust and upper mantle model. We delete the lower 2 layers of Gutenberg's model to make the 22-layer-model; 4 to make the 20-layer-model; and so on. The figure clearly shows that RTgen saves 55% computational time for the 4-layer-model; 57% for the 14-layer-model; 60% for the 24-layer-model. Tests on other computer platforms also show at least 50% savings on computational time.

NOTE ON LOVE WAVES

The same idea can be applied to Love waves, which consist of SH waves only. In another words we directly calculate the generalized R/T coefficients of Love waves without knowing the modified R/T coefficients.

For Love waves, the continuity condition at any arbitrary interface j states that (see Appendix B for each term)

$$\begin{bmatrix} E_{11}^{j} & E_{12}^{j} \\ E_{21}^{j} & E_{22}^{j} \end{bmatrix} \begin{bmatrix} \Lambda_d^{j}(z^{j}) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} C_d^{j} \\ C_u^{j} \end{bmatrix} = \begin{bmatrix} E_{11}^{j+1} & E_{12}^{j+1} \\ E_{21}^{j+1} & E_{22}^{j+1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \Lambda_u^{j+1}(z^{j}) \end{bmatrix} \begin{bmatrix} C_d^{j+1} \\ C_u^{j+1} \end{bmatrix} \qquad (12)$$

The definition of the generalized R/T coefficients implies that

$$\begin{cases} C_u^{j} = \hat{R}_{du}^{j} C_d^{j} \\ C_d^{j+1} = \hat{T}_d^{j} C_d^{j} \\ C_u^{j+1} = \hat{R}_{du}^{j+1} \hat{T}_d^{j} C_d^{j} \end{cases} \qquad (13)$$

where $\hat{R}_{du}^{j}$ and $\hat{T}_d^{j}$ are the generalized reflection and transmission coefficients of incident down-going SH-wave to reflected SH-wave and transmitted SH-waves at interface j, respectively. Directly substituting equation (13) into equation (12) yields

$$\begin{bmatrix} I \\ \hat{R}_{du}^{j} \end{bmatrix} = \begin{bmatrix} E_{11}^{j} & E_{12}^{j} \\ E_{21}^{j} & E_{22}^{j} \end{bmatrix}^{-1} \begin{bmatrix} E_{11}^{j+1} & E_{12}^{j+1} \\ E_{21}^{j+1} & E_{22}^{j+1} \end{bmatrix} \begin{bmatrix} \hat{T}_d^{j} \\ \Lambda_u^{j+1}(z^{j}) \hat{R}_{du}^{j+1} \hat{T}_d^{j} \end{bmatrix} \qquad (14)$$

Starting from the last interface where $\hat{R}_{du}^{N} = 0$, equation (14) yields the generalized reflection and transmission coefficients ($\hat{R}_{du}^{j}, \hat{T}_{d}^{j}$) of Love waves for all interfaces above. Appendix B gives explicit solutions for these coefficients.

Stability tests on Love waves show that above algorithm is correct and stable. However, efficiency tests show only a 1-5% speed-up. This is not a surprise due to the fact that inversion of 2X2 **E** matrix (equation (14)) is not major CPU time consuming part on SH-wave cases. Both traditional and our versions did equally well on calculation of inverse 2X2 **E** matrix.

CONCLUSIONS

Methods used to calculate surface wave dispersion curves for a flat-layered earth model have been undergone many improvements since the pioneering work of Thomson and Haskell. Among them, the methods based on R/T (reflection and transmission) coefficients produce stable phase velocities for both low and high frequencies and are appropriate for a viscoelastic and anisotropic media, if not the most efficient algorithms. This paper presents a new algorithm, called the fast generalized R/T coefficient method, to calculate the phase velocities of surface waves for a layered earth model.

Based on the generalized R/T coefficient method, the fast generalized R/T coefficient method calculates the generalized R/T coefficients without knowing the modified R/T coefficients. This is done by directly substituting the definition of the generalized R/T coefficients (equation (10)) into equation (9). The direct substitution results in a 4X4 layer matrix **E** of which all elements are determined by the elastic parameters of one layer. This characteristic allows many terms to be crossed out and results in a simple analytic 4X4 inverse matrix $\mathbf{E}^{-1}$.

Stability tests of the application on Rayleigh waves are performed by comparison of calculated dispersion curves computed by RTgen and by CPS, which is a popular free software package developed by Hermann and Ammon (2002). The dispersion curves cover over a broad range of frequency from 0.01Hz to 100 Hz. Both codes yield identical phase velocities to 1% accuracy for fundamental mode and 5% accuracy for the potentially exited higher modes, indicating that the fast generalized R/T coefficient method is accurate and stable. Efficiency tests are done on various computer platforms by comparison of computational time taken by the fast generalized R/T coefficient method (RTgen) and the traditional generalized R/T coefficient method (RTmod). The fast generalized R/T coefficient method saves at least 50% computational time, demonstrating its efficiency. Stability tests on Love waves show that the fast generalized R/T coefficient method is correct and stable. However, efficiency tests show only a 1-5% speed-up for Love wave dispersion calculation.

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TABLES

Table 1. Test models at crustal scale

Depth to bottom (km)		Density (g/cm3)		Vp (km/s)		Vs (km/s)	
M2*	M3	M2	M3	M2	M3	M2	M3
18	20	2.80	2.8	6.00	5.6	3.50	2.5
24	25	2.90	3.4	6.30	6.3	3.65	3.2
30	40	3.50	3.2	6.70	6.1	3.90	2.9
50	∞	3.40	3.4	6.00	6.3	3.70	3.2
∞		3.30		8.20		4.70	

*M2 means model 2; the same for M3

Table 2. Test models at local site scale

Depth to bottom (m)			Density (g/cm^3)			Vp (m/s)			Vs (m/s)		
M4*	M5	M6	M4	M5	M6	M4	M5	M6	M4	M5	M6
12	11	11	2.0	2.0	2.0	311.8	1558.8	866.0	180	900	500
24	23	23	2.0	2.0	2.0	519.6	866.0	1558.8	300	500	900
36	35	35	2.0	2.0	2.0	866.0	1212.4	1212.4	500	700	700
∞	130	130	2.0	2.0	2.0	1212.4	1732.0	1732.0	700	1000	1000
	∞	∞		2.0	2.0		2551.6	2551.6		1300	1300

*M4 means model 4; the same for M5 and M6

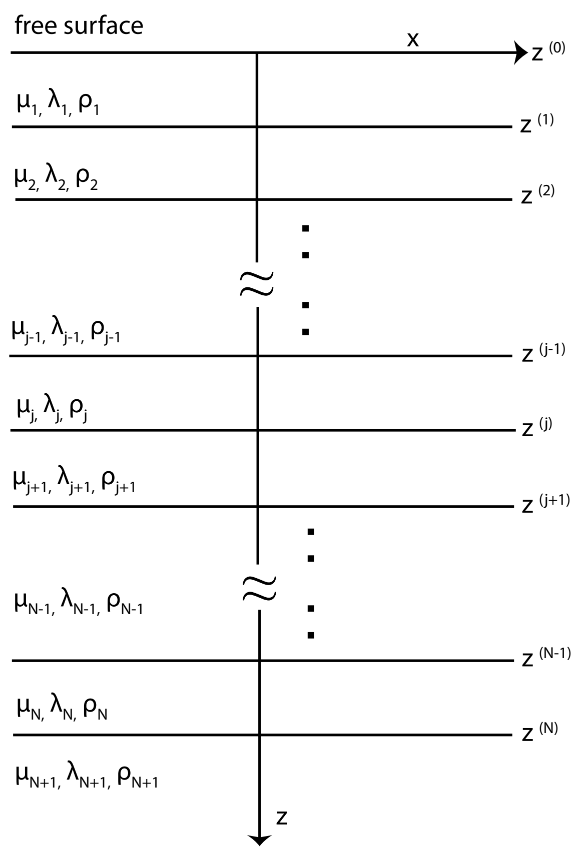
FIGURE CAPTIONS

Figure 1. Configuration and coordinate system of a multiple-layered half-space.

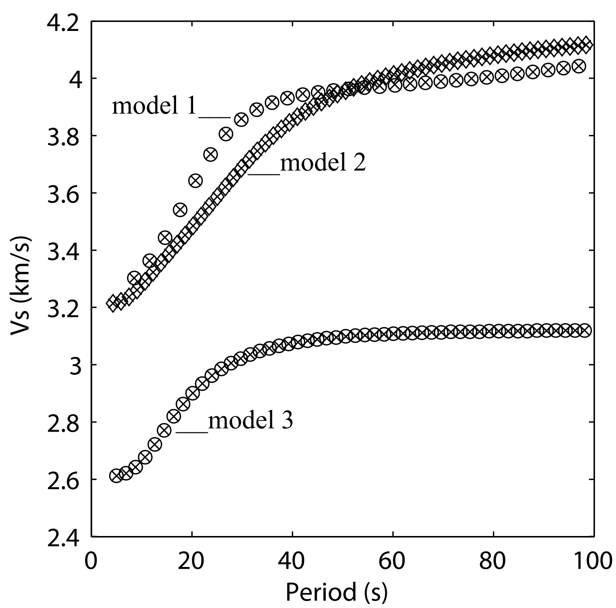
Figure 2. Phase velocity dispersion curves of the fundamental-mode Rayleigh waves for models 1, 2, and 3. The crosses are phase velocities calculated by RTgen. The circles are phase velocities calculated by CPS.

Figure 3. Phase velocity dispersion curves of the fundamental-mode Rayleigh waves for models 4, 5, and 6. The crosses are phase velocities calculated by RTgen. The circles and diamonds are phase velocities calculated by CPS. Note that models 5 and 6 are closely overlapped.

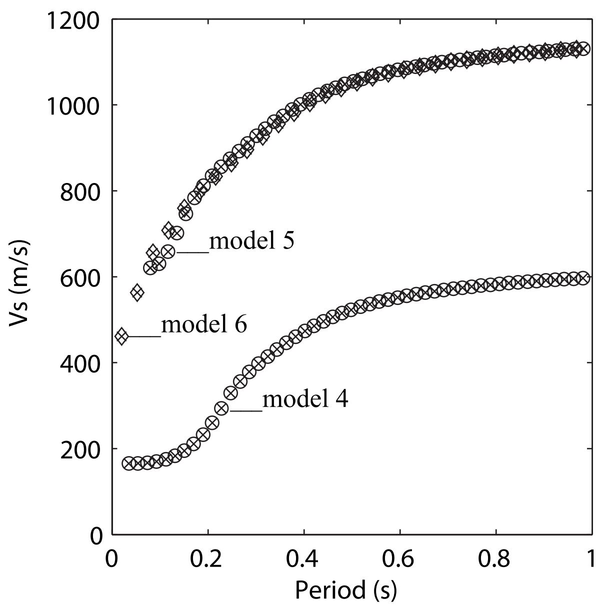
Figure 4. Computational time against number of layers in models. The solid line represents computational time taken by RTgen, and the dash line by RTmod. Clearly, the fast generalized R/T method cuts the computational time at least by half.



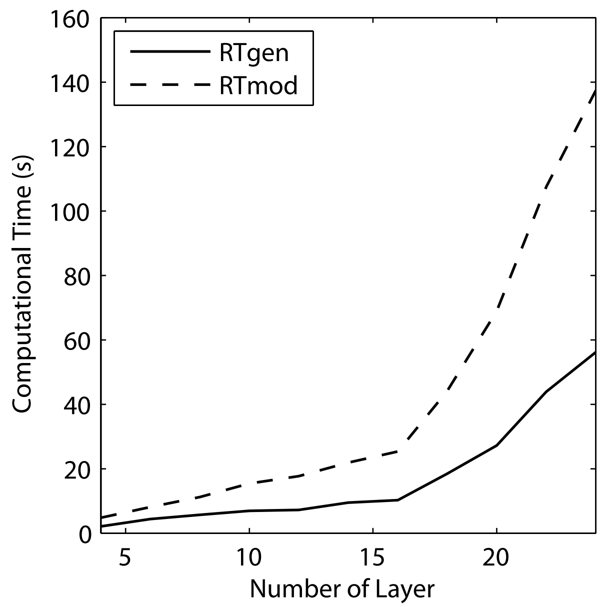
free surface
x
z (0)
μ1, λ1, ρ1
z (1)
μ2, λ2, ρ2
z (2)
μj-1, λj-1, ρj-1
z (j-1)
μj, λj, ρj
z (j)
μj+1, λj+1, ρj+1
z (j+1)
μN-1, λN-1, ρN-1
z (N-1)
μN, λN, ρN
z (N)
μN+1, λN+1, ρN+1
z



model 1
model 2
model 3
Vs (km/s)
Period (s)
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
0
20
40
60
80
100



1200
1000
800
600
400
200
0
Vs (m/s)
model 5
model 6
model 4
0
0.2
0.4
0.6
0.8
1
Period (s)


160
140
120
100
80
60
40
20
0
5
10
15
20
Number of Layer
Computational Time (s)
RTgen
RTmod